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AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

1. (Currently amended) A method for using a computer system to solve an unconstrained interval global optimization problem specified by a function *f*, wherein *f* is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \ldots x_n)$, the method comprising:

receiving a representation of the function *f* at the computer system;

storing the representation in a memory within the computer system; and

performing an interval global optimization process using interval operations to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$;

wherein performing the interval global optimization process involves,

applying term consistency over a subbox **X**, and

excluding any portion of the subbox **X** that violates term consistency.

2. (Original) The method of claim 1, wherein applying term consistency involves:

symbolically manipulating an equation within the computer system to solve for a first term, $g(\mathbf{x}')$, thereby producing a modified equation $g(\mathbf{x}') = h(\mathbf{x})$, wherein the first term $g(\mathbf{x}')$ can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

substituting the subbox **X** into the modified equation to produce the equation $g(\mathbf{X}') = h(\mathbf{X})$;

solving for $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$; and

intersecting $\mathbf{X}'$ with the subbox **X** to produce a new subbox $\mathbf{X}^+$;

wherein the new subbox $\mathbf{X}^+$ contains all solutions of the equation within the subbox **X**, and wherein the size of the new subbox $\mathbf{X}^+$ is less than or equal to the size of the subbox **X**.

3. (Original) The method of claim 1, wherein performing the interval global optimization process involves:

keeping track of a least upper bound *f_bar* of the function $f(\mathbf{x})$;

removing from consideration any subbox for which $f(\mathbf{x}) > f\_bar$;

applying term consistency to the inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox **X**; and

excluding any portion of the subbox **X** that violates the inequality.

4. (Original) The method of claim 1, wherein performing the interval global optimization process involves:

determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$;

removing from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the subbox does not include a global minimum of $f(\mathbf{x})$; and

applying term consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=0$ over the subbox **X**; and

excluding any portion of the subbox **X** that violates a component.

5. (Currently amended) The method of claim 1, wherein performing the interval global optimization process involves:

determining diagonal elements $H_{ii}(\mathbf{x})$ $(i=1,\ldots,n)$ of the Hessian of the function $f(\mathbf{x})$;

removing from consideration any subbox for which a diagonal element of the Hessian is always negative, which indicates that the f is not convex and consequently does not contain a global minimum within the subbox;

applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ $(i=1,\ldots,n)$ over the subbox **X**; and

excluding any portion of the subbox **X** that ~~violates an inequality~~violate these inequalities.

6. (Currently amended) The method of claim 1, wherein performing the interval global optimization process involves performing the Newton method, wherein performing the Newton method involves:

computing the Jacobian $\mathbf{J}(\mathbf{x},\mathbf{X})$ of the function f evaluated as a function of a point **x** over the subbox **X**;

computing an approximate inverse **B** of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$; and

using the approximate inverse **B** to analytically determine the system $\mathbf{Bg}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$~~; and~~

~~applying term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ $(i=1,\ldots,n)$ for each variable x_i $(i=1,\ldots,n)$ over the subbox **X**; and~~

~~excluding any portion of the subbox **X** that violates a component~~.

7. (Original) The method of claim 1, further comprising terminating attempts to further reduce the subbox **X** when:

the width of **X** is less than a first threshold value; and

the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

8. (Currently amended) A computer-readable storage medium storing instructions that when executed by a computer cause the computer to perform a method for using a computer system to solve an unconstrained interval global optimization problem specified by a function f, wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \ldots x_n)$, the method comprising:

receiving a representation of the function f at the computer system;

storing the representation in a memory within the computer system; and

performing an interval global optimization process using interval operations to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$;

wherein performing the interval global optimization process involves,

applying term consistency over a subbox $\mathbf{X}$, and

excluding any portion of the subbox $\mathbf{X}$ that violates term consistency.

9. (Original) The computer-readable storage medium of claim 8, wherein applying term consistency involves:

symbolically manipulating an equation within the computer system to solve for a first term, $g(\mathbf{x}')$, thereby producing a modified equation $g(\mathbf{x}') = h(\mathbf{x})$, wherein the first term $g(\mathbf{x}')$ can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

substituting the subbox $\mathbf{X}$ into the modified equation to produce the equation $g(\mathbf{X}') = h(\mathbf{X})$;

solving for $\mathbf{X}' = g^{-1}(h(\mathbf{X}))$; and

intersecting $\mathbf{X}'$ with the subbox $\mathbf{X}$ to produce a new subbox $\mathbf{X}^+$;

wherein the new subbox $\mathbf{X}^+$ contains all solutions of the equation within the subbox **X**, and wherein the size of the new subbox $\mathbf{X}^+$ is less than or equal to the size of the subbox **X**.

10. (Original) The computer-readable storage medium of claim 8, wherein performing the interval global optimization process involves:

keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$;

removing from consideration any subbox for which $f(\mathbf{x}) > f\_bar$;

applying term consistency to the inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox **X**; and

excluding any portion of the subbox **X** that violates the inequality.

11. (Original) The computer-readable storage medium of claim 8, wherein performing the interval global optimization process involves:

determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$;

removing from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the subbox does not include a global minimum of $f(\mathbf{x})$; and

applying term consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=0$ over the subbox **X**; and

excluding any portion of the subbox **X** that violates a component.

12. (Currently amended) The computer-readable storage medium of claim 8, wherein performing the interval global optimization process involves:

determining diagonal elements $H_{ii}(\mathbf{x})$ $(i=1,\ldots,n)$ of the Hessian of the function $f(\mathbf{x})$;

removing from consideration any subbox for which a diagonal element of the Hessian is always negative, which indicates that the f is not convex and consequently does not contain a global minimum within the subbox;

applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$; and

excluding any portion of the subbox $\mathbf{X}$ that ~~violates an inequality~~violate these inequalities.

13. (Currently amended) The computer-readable storage medium of claim 8, wherein performing the interval global optimization process involves performing the Newton method, wherein performing the Newton method involves:

computing the Jacobian $\mathbf{J}(\mathbf{x},\mathbf{X})$ of the function f evaluated as a function of a point $\mathbf{x}$ over the subbox $\mathbf{X}$;

computing an approximate inverse $\mathbf{B}$ of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$; and

using the approximate inverse $\mathbf{B}$ to analytically determine the system $\mathbf{Bg}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$~~; and~~

~~applying term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ $(i=1,\ldots,n)$ for each variable x_i $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$; and~~

~~excluding any portion of the subbox $\mathbf{X}$ that violates a component~~.

14. (Original) The computer-readable storage medium of claim 8, wherein the method further comprises terminating attempts to further reduce the subbox $\mathbf{X}$ when:

the width of $\mathbf{X}$ is less than a first threshold value; and

the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

15. (Currently amended) An apparatus that solves an unconstrained interval global optimization problem specified by a function *f*, wherein *f* is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \ldots x_n)$, the apparatus comprising:

a receiving mechanism that is configured to receive a representation of the function *f*;

a memory for storing the representation; and

an interval global optimization mechanism that is configured to perform an interval global optimization process <u>using interval operations</u> to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$;

a term consistency mechanism within the interval global optimization mechanism that is configured to,

apply term consistency over a subbox $\mathbf{X}$, and to

exclude any portion of the subbox $\mathbf{X}$ that violates term consistency.

16. (Original) The apparatus of claim 15, wherein the term consistency mechanism includes:

a symbolic manipulation mechanism that is configured to symbolically manipulate an equation within the computer system to solve for a first term, $g(\mathbf{x'})$, thereby producing a modified equation $g(\mathbf{x'}) = h(\mathbf{x})$, wherein the first term $g(\mathbf{x'})$ can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

a solving mechanism that is configured to,

substitute the subbox $\mathbf{X}$ into the modified equation to produce the equation $g(\mathbf{X'}) = h(\mathbf{X})$, and to

solve for $\mathbf{X'} = g_1^{-1}(h(\mathbf{X}))$; and

an intersecting mechanism that is configured to intersect $\mathbf{X'}$ with the subbox $\mathbf{X}$ to produce a new subbox $\mathbf{X}^+$;

wherein the new subbox $\mathbf{X}^+$ contains all solutions of the equation within the subbox **X**, and wherein the size of the new subbox $\mathbf{X}^+$ is less than or equal to the size of the subbox **X**.

17. (Original) The apparatus of claim 15,

wherein the interval global optimization mechanism is configured to,

keep track of a least upper bound *f_bar* of the function $f(\mathbf{x})$, and to

remove from consideration any subbox for which $f(\mathbf{x}) > f\_bar$;

wherein the term consistency mechanism is configured to,

apply term consistency to the inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox **X**, and to

exclude any portion of the subbox **X** that violates the inequality.

18. (Original) The apparatus of claim 15,

wherein the interval global optimization mechanism is configured to,

determine a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$, and to

remove from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the subbox does not include a global minimum of $f(\mathbf{x})$; and

wherein the term consistency mechanism is configured to,

apply term consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=0$ over the subbox **X**, and to

exclude any portion of the subbox **X** that violates a component.

19. (Currently amended) The apparatus of claim 15, wherein the interval global optimization mechanism is configured to,

determine diagonal elements $H_{ii}(\mathbf{x})$ (i=1, ..., n) of the Hessian of the function $f(\mathbf{x})$, and to

remove from consideration any subbox for which a diagonal element of the Hessian is always negative, which indicates that the f is not convex and consequently does not contain a global minimum within the subbox; and

wherein the term consistency mechanism is configured to,

apply term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ (i=1, ..., n) over the subbox **X**, and to

exclude any portion of the subbox **X** that ~~violates an inequality~~violate these inequalities.

20. (Currently amended) The apparatus of claim 15, further comprising a Newton mechanism within the interval global optimization mechanism that is configured to:

compute the Jacobian **J**(**x**,**X**) of the function f evaluated as a function of a point **x** over the subbox **X**;

compute an approximate inverse **B** of the center of **J**(**x**,**X**); and to

using the approximate inverse **B** to analytically determine the system **Bg**(**x**), wherein **g**(**x**) is the gradient of the function $f(\mathbf{x})$, and wherein **g**(**x**) includes components $g_i(\mathbf{x})$ (i=1, ..., n)~~;~~

~~wherein the term consistency mechanism is configured to,~~

~~apply term consistency to each component ($\mathbf{Bg}(\mathbf{x}))_i = 0$ (i=1, ..., n) for each variable x_i (i=1, ..., n) over the subbox **X**, and to~~

~~exclude any portion of the subbox **X** that violates term consistency~~.

21. (Original) The apparatus of claim 15, further comprising a termination mechanism that is configured to terminate attempts to further reduce the subbox **X** when:

the width of **X** is less than a first threshold value; and

the magnitude of $f(\mathbf{X})$ is less than a second threshold value.